Name: **2008 Summer II** **M463**
Exam 1

Numerical expressions may be left unsimplified, but I expect you to do sums of a geometric series.

1. Suppose that 30% of IU Finite students have taken calculus in high school. Of those who had calculus in high school, 80% get an A in Finite; of those who have not taken calculus in high school only 15% get an A. Suppose a randomly selected Finite student happens to have gotten an A. What is the probability s/he took calculus in high school?

2. Suppose a fair die is rolled 9 times. Let E be the event that a six came up on at least one of rolls, and let F be the event that the *first* six came up on an odd roll, i.e. rolls 1, 3, 5, 7, or 9.

   (a) $P(E) = ?$ $= 1 - P(\text{no six}) = 1 - \left(\frac{5}{6}\right)^9$ ✓ P(at least one six ∩ first six on an odd roll)

   (b) $P(E \cap F) = ?$ Using geometric

   (c) $P(F|E) = ?$ $\frac{P(E \cap F)}{P(E)}$

3. There's a carnival game that consists of attempting to toss a roll of toilet paper into a toilet bowl. (Really.) Suppose you have a 90% chance of succeeding at this on a single trial.

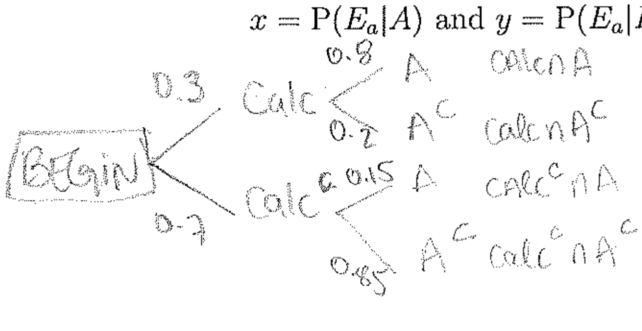
   (a) Use the Bernoulli formula to calculate the exact probability that you get at least two successes in five trials.

   (b) Use the Normal approximation to estimate the probability of getting at least 20 successes in 50 trials. You may express your answer in terms of the Φ function, i.e. you may leave expressions like $\Phi(1.3)$ unevaluated.

4. How large a sample size would be needed if a pollster wants to be 95% confidant that the sample percentage of Indiana residents who like goat cheese is within 1% of the true percentage?

5. A rat has a two room apartment in a Biology lab. Let us denote the rooms by a and b, and let A be the event that the rat is currently in room a, and let B be the event that it is in room b. Also suppose that rooms a and b each have their own exits from the apartment, and let E_a be the event that the rat eventually exits the apartment by the room a exit, and E_b be the event it eventually exits by room b. Assume that if the rat is currently in room a there is a 30% chance it will exit immediately by the a-exit and a 70% chance that it will move to room b instead, and if the rat is currently in room b there it is equally likely immediately leave through the b-exit as to move to room a. Calculate $P(E_a|A)$ and $P(E_a|B)$.

   Hint: Use conditioning on the rat's first move to set up a system of equations in $x = P(E_a|A)$ and $y = P(E_a|B)$.



$P(\text{calc} \mid A) = \frac{P(\text{calc} \cap A)}{P(A)}$

$= \frac{0.3 \times 0.8}{0.3 \times 0.8 + 0.7 \times 0.15} = \frac{0.24}{0.24 + 0.105}$

$= \frac{0.24}{0.345} = \boxed{0.6956}$ ✓

(a) $P(\text{at least 2 in 5}) = 1 - P(0 \text{ or } 1) = 1 - \binom{5}{0}(.9)^0(.1)^5 - \binom{5}{1}(.9)^1(.1)^4$

$= 1 - (.1)^5 - 5(.9)^1 \times (.1)^4$

$1 - 0.00001 - 0.00045 = \boxed{0.99954}$

(b) $P(\text{at least 20 in 50})$

$X \sim Bin(50, .9)$ $\mu = n \cdot p = 50 \times .9 = \boxed{45}$

$= P(X \geq 20)$

$\sigma = \sqrt{npq} = \sqrt{50 \cdot (.9)(.1)} = \sqrt{4.5} = 2.12132$

$= 1 - P(X < 20) = 1 - P(X \leq 19) = 1 - P\left(\frac{X - \mu}{\sigma} \leq \frac{19 - 45}{\sqrt{4.5}}\right)$

$\approx 1 - P\left(Z \leq \frac{19.5 - 45}{\sqrt{4.5}}\right) = 1 - P\left(Z \leq \frac{-25.5}{\sqrt{4.5}}\right) = 1 - P(Z \leq -12.0208)$

$= P(Z \leq 12.0208) = \boxed{\Phi(12.0208)}$

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let X = # of people that like g.c.
n = sample size

$\bar{X} = \frac{X}{n}$

p = true percentage $= \frac{G}{N}$

$E = |\bar{X} - p| \leq 0.01$ E is the event that the sample percentage is within 1% of the true percentage.

We want

$P(|\bar{X} - p| \leq 0.01) = 0.95. \Rightarrow P(-0.01 \leq \bar{X} - p \leq 0.01) = 0.95$

$\sigma \leq \frac{\sqrt{n}}{2}$

$P(-0.01 \cdot n \leq X - np \leq 0.01n) = 0.95$

$P\left(\frac{-0.01 \cdot n}{\sigma} \leq \frac{X - \mu}{\sigma} \leq \frac{0.01n}{\sigma}\right) = 0.95$

$\approx P\left(\frac{-0.01 \times n}{\sigma} \leq Z \leq \frac{0.01n}{\sigma}\right) = 0.95$

$P\left(\frac{-0.01n}{\sigma} \leq Z \leq \frac{0.01n}{\sigma}\right) \geq P\left(\frac{-0.01n}{\sqrt{n}/2} \leq Z \leq \frac{0.01n}{\sqrt{n}/2}\right) = .95$

$2\Phi(z_0) - 1 = .95$, where $z_0 = 2 = \frac{0.01n}{\sqrt{n}/2} = 2 = \frac{2 \times 0.01n}{\sqrt{n}}$

$2\sqrt{n} = 0.02 \cdot n \Rightarrow \sqrt{n} = 0.01n \Rightarrow \boxed{n = 10{,}000}$