13

MATH 343 Summer I 2013 August 13^{th}, 2013
Aseel Farhat

Quiz#1 **Name:** Enrique Areyan

You have 20 minutes to finish the following 3 problems.

1. (*5 points*) Solve the following I.V.P.

$$\frac{y'}{t} + \frac{y}{t^2} = 5, \qquad y(1) = 1.$$

(1) Standard form: (multiply by t)

$$y' + \frac{y}{t} = 5t.$$

Note that by Uniqueness and Existence THEOREM, since $p(t) = \frac{1}{t}$ and $g(t) = 5$ are continuous everywhere except in $t=0$ for $p(t)$, AND $t_0 = 1$, we will have a unique solution on $(0, \infty)$. Solve the eq by integrating factor $\mu(t) = e^{\int \frac{1}{t} dt} = e^{\ln(t)}$

$\Rightarrow \mu(t) = t.$

(2) multiply both sides by $\mu(t)$: $t\left[y' + \frac{y}{t}\right] = 5t^2$

(3) By product rule: $\frac{d}{dt}[ty] = 5t^2$

(4) Integrate both sides $\int \frac{d}{dt}[ty]\,dt = \int 5t^2\,dt$

(5) solve:

$ty = \frac{5}{3}t^3 + C \Rightarrow \boxed{y = \frac{5}{3}t^2 + \frac{C}{t}}$ this is the general sol. Finally, solve for C:

$y(1) = 1 = \frac{5}{3} + C \Rightarrow C = 1 - \frac{5}{3} \Rightarrow \boxed{C = \frac{-2}{3}}$ the particular solution is $\boxed{y(t) = \frac{5}{3}t^2 - \frac{2}{3t}}$

2. (*5 points*) State the existence and uniqueness theorem for linear first order differential equations and determine where the solution of the given I.V.P. is certain to exist and be unique.

$$(t-4)y' + \ln(t-1)y = \frac{t^2}{t-6}, \qquad y(2) = 6.$$

The existence and uniqueness theorem.

Consider the I.V.P. (*): $\begin{cases} y' + p(t)y = g(t) \\ y(t_0) = y_0 \end{cases}$

If $p(t)$ and $g(t)$ are continuous over and interval $\alpha < t < \beta$

AND $t_0 \in (\alpha, \beta)$.

THEN there exists a unique solution $\phi(t)$ to (*).

Consider the equation above. to use THEOREM, first write in standard form (multiplying by $(t-4)^{-1}$):

$$y' + \frac{\ln(t-1)}{t-4} y = \frac{t^2}{(t-4)(t-6)}.$$

Now, let $p(t) = \frac{\ln(t-1)}{t-4}$ AND $g(t) = \frac{t^2}{(t-4)(t-6)}$ the function

$P(t)$ is continuous if $t > 1$ (since $\ln(t-1)$ cannot be negative) AND $t \neq 4$

$g(t)$ is continuous if $t \neq 4$ AND $t \neq 6$.

Now, by U.E.T., given that our initial conditie is $t_0 = 2$, we will have a solution (unique) if $\boxed{t \in (1,4)}$

3. (*5 points*) Find the general solution of the following differential equation:

$$y' + 2y = 7ty^2.$$

this is a 1^{ST} O.D.E, non linear, fitting the Bernoulli CASE when $n=2$. It is already in standard form.

we make the change: $u = y^{1-n} = y^{-1}$

$$u' = -1\, y^{-2} y' \Rightarrow y' y^{-2} = -u'$$

Multiply the original eq. by y^{-2}:

$$y'y^{-2} + 2y^{-1} = 7t$$

Make the change

$$-u' + 2u = 7t$$

Now, this is a 1^{st} O.D.E. linear. Solve by integrating facto

(1) Write in standard form: $u' - 2u = -7t$

(2) integrating factor: $e^{\int -2dt} = e^{-2t}$

(3) $e^{-2t}[u' - 2u] = -7te^{-2t}$

(4) $\int \frac{d}{dt}[e^{-2t}u] = \int -7te^{-2t}\,dt$

(5) $e^{-2t}u = -7\int_{t_0}^{t} se^{-2s}\,ds + C$

$$u = \left(-7\int_{t_0}^{t} se^{-2s}\,ds + C\right)/e^{-2t}$$

(6) Change back to y: $u = y^{-1}$

$$\Rightarrow \boxed{y(t) = \frac{e^{-2t}}{-7\int_{t_0}^{t} se^{-2s}\,ds + C}}$$

-2