Homework 3 for M312, Section 30353
due Wednesday, September 18, 2013

1. (10 pts) Prove the formula $\Delta(fg) = f\Delta g + 2\nabla f \cdot \nabla g + g\Delta f$.
2. (10 pts) Prove the formula $div(f\nabla g - g\nabla f) = f\Delta g - g\Delta f$.
3. (10 pts) For $(x, y) \in \mathbb{R}^2$, $(x, y) \neq (0, 0)$, let $f(x, y) = \log(x^2 + y^2)$. Compute Δf.
4. (10 pts) Show that for any $v, w \in \mathbb{R}^3$ one has $\sqrt{||v||^2||w||^2 - (v \cdot w)^2} = ||v \times w||$.
5. (10 pts) Compute the curvature of the path $c(t) = (\cos t, \sin t, t^2)$ at arbitrary t.
6. (20 pts) Find an appriopriate parametrization for the curve which is the intersection of the surfaces $y = x$ and $z = x^2$ from the point $(-2, -2, 4)$ and $(1, 1, 1)$. Find the total curvature of this curve.
7. (10 pts) Show that the work done by the gravitational vector field in $\mathbb{R}^3$ centered at the origin (with $G = m = M = 1$) as a particle moves from point p to point q depends only $||p||$ and $||q||$.
8. (20 pts) Compute $\displaystyle\int_c \frac{xdx + ydy}{x^2 + y^2}$, where $c(t) = (e^t, t^2)$, $0 \leq t \leq 1$.